Should the UK Join the Euro Zone?
Evidence from a Synthetic OCA Assessment
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Abstract:

Based on optimal currency area theories, this paper applies the empirical methodology of structural VAR to revisit the question of whether UK should join or not the Euro zone. Correlations of effective exchange rates between UK and US on one side and between UK and its major European partners (Germany and France) on the other side are first estimated. Correlations between macro-structural shocks are then estimated and results of both approaches are combined to provide conclusions on the appropriateness of a floating regime of the British Pound or of the integration to the Euro zone. A dynamic analysis shows that UK structural evolution over the period 1970-2008 tends to favor the option of integration to the Euro zone. These results have to be qualified by taking into account the specific position of UK with London as a global financial center and political issues which dominate the debate.

JEL codes: F15, F31, F36

Keywords: Sterling Pound, Euro, Optimal Currency Area

1. Introduction

The question of whether UK will join the Euro zone, at some stage, is predominantly a political issue. It is debated in political programs periodically and settled in pools. The historical alliance with the US, the singularity of the City as a global financial center explained the reluctance to join the European Union to begin with, and then to participate to the process of monetary union achieved with the adoption of the Euro.

From a strictly economical point of view, the cost/benefit analysis of the joining of UK to the Euro zone has been thoroughly investigated in the so-called ‘Euro report’ commissioned by H.M. Treasury (2003), concluding prudently in favor of the ‘Canadian solution’. The example of Canada apparently shows that a relatively small economy need not form a monetary union with a large and very close neighbor (i.e. the US) and can manage macro financial policies with a floating exchange rate regime. The same would consequently go for the UK with the Euro zone.

A survey by Artis (2006) acknowledges the predominantly political nature of the issue, but somewhat mitigates the Treasury position by a counter examination of the report’s discussion of optimal currency area (OCA)

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arguments that had been used to reach a negative conclusion. In particular, the conclusion that exchange rate flexibility would be a necessary adjustment mechanism to absorb asymmetric shocks is challenged.

Another contribution by Pesaran et al. (2006) shows also that the joining of UK to the Euro zone could have positive effects in terms of output growth, interest rates and reduction of inflation, although results are dependent on the rate of exchange chosen for the entry and on the date of entry.

On methodological grounds, the OCA analysis has been enriched by the use of macro econometric methods, such as global VAR (GVAR) or structural VAR (SVAR), providing new estimates on the effects for UK of joining or staying outside (Artis 2006), with no clear-cut conclusions.

Obviously the question remains open for future debates. Moreover, recent financial and monetary disturbances affected both economies members of the Euro zone and outsiders, whether willing to join or to remain outside. Members of the Euro zone, deprived of the adjustment mechanism by an exchange rate of a national currency are severely constrained in the conduct of their macroeconomic policies, while on the other hand European countries outside the Euro zone experience dangerous instability of their exchange rates, menaces of uncontrollable speculation and risks of disruption of their links with countries of the Euro zone to which they are economically closely integrated.

The dilemma between independence with instability and a strait jacket providing a more stable environment is present more than ever. The menace of a cumulative process of competitive devaluations increases uncertainty and probably justifies the willingness to participate to a bloc of stability, as provided by the Euro zone, even though it is surrounded by instability in its external relationships.

Yet from an economic point of view, integration to a monetary union has also to be judged in terms of costs of adjustment born by the joining country when structural characteristics of both parts are too dissimilar. In fact the OCA theories often consider economic convergence as a prerequisite for regional monetary integration. In recent approaches based on economic shocks’ characteristics, an OCA is defined as an economic block consisting of countries affected symmetrically by shocks. If this condition is not fulfilled, according to OCA approaches, it is exchange rate flexibility that absorbs asymmetric shocks. In this sense, observed exchange rate variability can be considered as a standard measure of the intensity of these shocks - von Hagen & Neumann (1994), De Grauwe & Heens (1993), Vaubel (1976). In other words, with free floating, a relative stability of observed exchange rate between two blocs (here UK and the Euro zone) would be an a priori indicator of the viability of the enlargement, while an important volatility of the observed exchange rate would reveal the necessity of exchange rate flexibility as adjustment tool.
By looking only at exchange rates behavior one is yet probably missing essential structural characteristics that can be blurred by discretionary exchange rate policies in contradiction with macro-structural shocks. In order to assess the viability, from an OCA methodology point of view, of the joining of UK to the Euro zone, we therefore propose an empirical approach combining two analyses:

- We observe on the long run (1970-2008) the characteristics of the Sterling Pound (GBP) effective exchange rate, as compared to the effective exchange rate of the monetary unit of the main commercial partners of UK, i.e. USA (USD) on the one hand, Germany and France (DM and then Euro, FF and then Euro) on the other hand. By using correlation method we measure the degree of co-movement between GBP and each of these three monetary units. These results, assumed to reflect indirectly symmetric or asymmetric structural shocks, can in fact be in discordance with the direct measure of the phenomenon.

- We consequently propose to measure directly structural shocks over the same period, by using the structural vector autoregressive (SVAR) model proposed originally by Blanchard & Quah (1989) and comparing likewise co-movement of structural shocks between UK and its main partners.

Comparison of both approaches provides information on the appropriateness of free floating of the Sterling or stabilization either with the USD or eventual fixity with the monetary unit of its two major continental partners (Germany and France), which means presently integration to the Euro zone.

Over the period 1970-2008, the share of goods and services traded (X+M) with the USA in UK total trade declined from 15% to 11%, while the shares of trade with Germany and France increased respectively from 5.5% to 13.5% and from 4% to 8%. This indicates obviously a structural evolution disconnecting to some extent UK from its traditional link with the US and deepening integration with continental Europe, as represented by UK’s major European partners. But it remains to confirm that macro-structural characteristics of both UK and its continental partners followed an evolution compatible with the joining of UK to the Euro zone.

In section 2 we will discuss the methodology proposed to assess dynamically the evolution of structural characteristics of UK as compared to its main trade partners over the period 1970-2008. Empirical results are presented and discussed in section 3. Section 4 concludes on the contribution of OCA methodology to the debate over the joining of UK to the Euro zone, these conclusions requiring qualifications to take into accounts political issues, as well as economic dimensions not being dealt with by OCA approaches.
2. Methodology

In order to examine if the UK would better join the Euro from an economic viewpoint, we measure the degree of disconnection of the Sterling Pound behavior from the UK’s economic fundamentals. To measure this disconnection, correlation coefficients are estimated between variables of the UK on the one hand, the USA, Germany and France on the other hand. Based on these correlation coefficients, a synthetic approach is then presented. These steps being described later in this section, the correlation methodology used in this paper should be specified first.

In this paper, correlation coefficients are used to measure co-movement between pairs of effective exchange rates, but also in order to assess similarity of macro-structural shocks. Among several correlation coefficients, Pearson’s is often used. But this coefficient does not fit our present analysis, because it imposes the hypothesis of normal distribution of the series, which for exchange rate series is not likely to be the case generally. To avoid imposing an inappropriate hypothesis for our series, Spearman’s rank-ordered correlation is preferred. To estimate this type of correlation, first, the value of each observation is reclassified in terms of rank. The differences between these ranks, $D$, and the number of observation pairs, $N$, are then used to estimate the correlation coefficient. Then, significance level is tested by a $t$-statistics $t = \rho \sqrt{\frac{N-2}{N-1}}$ which follows a distribution of Student with a degree of freedom equal to $(N-2)$, under the null hypothesis of zero correlation. For example, Spearman’s correlation coefficient between a variable of the UK and that of Germany, $\rho_{uk,bd}$, can be expressed as follows:

$$\rho_{uk,bd} = 1 - \frac{6 \sum D^2}{N(N^2 - 1)}$$  

(1)

where $D$ represents the difference between the two ranks of the corresponding values of the UK’s and Germany’s variables, while $N$ indicates the number of observation pairs.

This rank-ordered correlation is then estimated in a dynamic approach: A sliding correlation is used, instead of estimating only one coefficient covering all the examined period. In this paper, one correlation coefficient is obtained for a period of 10 years, and the period slides, quarter by quarter, from 1970Q1 to 1980Q4.

To check the robustness of our results, we estimated correlation coefficients for 8 years, 10 years and 12 years. As these three lengths did not give significantly different results, we finally retained 10-year based correlation in order to represent the long run.
2008Q3. This approach allows to observe evolutions of the examined correlation and its dynamic adjustments over time.

2.1 Characteristics of exchange rate behavior

The first step of the analytical framework is to examine UK nominal effective trade-weighted exchange rate co-variation behavior with its three main trade partners, mentioned above. The objective of this analysis is to check if there is a currency to which the GBP is more closely linked, despite its free-floating exchange rate regime. In general, one can consider two methods aiming at determining if a currency is related to another one. The first one consists in examining stability of the nominal bilateral exchange rate between the two currencies. If the rate is stable, one can argue that the two currencies are closely related to each other. But on the contrary, if the rate’s volatility is high, the two currencies are considered unrelated to each other. Although this method seems to be useful, there are some shortcomings in its use. First, it is very difficult to quantify the degree of stability or that of volatility. Furthermore, it is often ambiguous to interpret correctly results on the stability or volatility because of measure-unit-related problems.

The second method is to analyze co-varying characteristics between two currencies’ nominal effective exchange rates. In this approach, one can argue that the more the two effective exchange rates are co-varying, the more the two currencies are closely related to each other. This method seems particularly useful in analyzing two currencies’ co-varying behavior when they are not fixed to each other but related to each other to some extent, and when each of them has its own bilateral links with other currencies, of which the combination is too complicated to take into account precisely for each of the two currencies. One main difficulty in using this method however lies in calculating exactly the most-widely used effective exchange rates, the trade-weighted one, which should take into account all the time-varying weights of each trade partners. In this paper we prefer the second method, combined with Spearman’s correlation, in order to examine if the GBP has some co-varying characteristics with currencies of its main trade partners: the higher the correlation, the stronger its nominal exchange rate’s link.

2.2. Structural characteristics

This step consists in characterizing economic fundamentals of the UK, to which the GBP exchange rate behavior will be compared. In an OCA approach, exchange rate flexibility is considered as the principal adjustment mechanism to economic shocks affecting countries in an asymmetric way. Thus, characteristics of these shocks and exchange rate variability are at the core of an OCA analysis. In this paper, essential macro-structural shocks to the UK and its main trade partners will be estimated and characterized for an assessment of the similarity of their economic conditions. For the UK, the nature of these shocks could clarify
the necessity of its exchange rate flexibility, or on the contrary, could plead in favor of joining the Euro.

Yet macro-structural shocks not being directly observable, it should be estimated empirically. In order to identify and recover these series of shocks to each of the examined countries, we use the structural VAR system proposed by Blanchard & Quah (1989) and extended by Clarida & Gali (1994). Technical details on the SVAR model being presented in appendix we describe here only some key elements of our methodology as follows. We construct first a standard 3-variable VAR model with real GDP (GDP), real effective exchange rate (REX) and consumer price index (CPI), for each of the examined countries. With $\Delta$ and $L$ as the usual difference and lag operators, respectively, the standard VAR system can be written in a moving average form as follows:

\[
X_t = B(L)e_t
\]  

(2)

where: $X =$ vector of the model’s 3 variables (non-stationary in levels but stationary in first differences\(^7\)), i.e., $X_t=[\Delta GDP, \Delta REX, \Delta CPI]'$; $e_t =$ vector of the errors in the standard VAR, i.e., $e_t=[e_{\Delta GDP}, e_{\Delta REX}, e_{\Delta CPI}]'$; $B_i =$ square 3x3 matrices of coefficients with $\sum_{i=0}^{k}B_iL^i$. This system can be easily estimated by the usual OLS method.

Then, the system can be expressed as a moving average function of white noise disturbances which correspond to three types of macro-structural shocks: aggregate supply (AS) shocks, real demand (IS) shocks and monetary demand (LM) shocks. This structural VAR system can be written as follows:

\[
X_t = C(L)e_t
\]  

(3)


\(^6\) In our approach, real effective exchange rate is included as a ‘global’ open-economy variable which covers all impacts of foreign variables on domestic ones, in particular in terms of output and prices. This ‘one-country’ model, estimated for each of the examined countries, is specifically preferred to examine the similarity of economic fundamentals in the following steps of our methodology based on international correlations.

\(^7\) As indicated in the following section of empirical results, each of the 3 variables of our model is characterized as I(1) but not co-integrated with each other.
where: $\varepsilon_t =$ vector of white noise disturbances, i.e., $\varepsilon_t = [\varepsilon_{AS}, \varepsilon_{IS}, \varepsilon_{LM}]'; C_i =$ square 3x3 matrices of coefficients with $Q(\lambda_L) = \sum_{i=0}^{\infty} C_i \lambda_i^i$. Based on information from estimates of the standard VAR and following Blanchard & Quah (1989), we use the relationship $\{ \varepsilon_t = C_0^{-1} \varepsilon_t \}$ in order to recover series of macro-structural shocks. But to this end, we need to correctly identify the 3x3 matrix $C_0$, which requires three theoretical and additional restrictions. As identification restrictions, we assume, as in Clarida & Gali (1994) as well as in many other macro models, that: (1) real and monetary demand shocks do not have any impact on the level of real GDP in the long run; and (2) monetary demand shocks do not have any impact on the level of real exchange rates in the long run.

Once the series of AS, IS and LM shocks estimated for each of the examined countries, the similarity of economic fundamentals between UK and the other countries will be measured by correlation coefficients. For the three bilateral relations, correlation coefficients will be computed for each of the three pairs of shocks. This will allow to discuss in detail similarity of aggregate supply side fundamentals (AS shock symmetry), that of real demand side (IS shock symmetry) as well as that of monetary demand side (LM shock symmetry).

If the UK is experiencing symmetric shocks with another country, correlations of shocks will be close to 1. In this case, ‘fluctuations’ of the GBP vis-à-vis the partner country’s currency are not required, and the expected ‘co-movement’ of the two currencies would be plausible. The two countries will be then considered as forming an OCA, which is an economic block consisting of countries affected symmetrically by shocks. But if the GBP is not co-varying with its partner’s currency, while macro-structural shocks are symmetric, the GBP behavior will be considered as disconnected from economic fundamentals.

2.3. Synthesis: Indicator of mismatch

The OCA theories assume that exchange rate variations absorb asymmetric shocks. Then, important exchange rate variability should imply that shocks are asymmetric, while exchange rate stability might indicate that shocks are symmetric. If this OCA assumption holds, the bilateral exchange rate behavior fits economic fundamentals. But, what if exchange rate is stable while economic shocks are asymmetric? Thus, if the OCA assumption does not hold, it is clear that there is exchange rate behavior mismatch with economic fundamentals.

In order to test this assumption and examine the compatibility of the GBP behavior with UK’s economic fundamentals, the next step of the analytical framework consists in synthesizing the characteristics of nominal effective exchange rate and those of macro-structural shocks described in the previous sections. To simplify this synthesis, it is possible to use an indicator measuring

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8 Cf. Appendix.
the difference between exchange rate co-movement and economic shock symmetry. This allows to quickly observe degree of incompatibility between those two characteristics of the economy. The indicator is defined as follows:

$$\text{Indicator of mismatch} = \rho_n - \rho_r$$

(4)

where: $\rho_n =$ ‘nominal’ correlation (co-movement) of nominal effective exchange rates; $\rho_r =$ ‘real’ correlation (symmetry) of macro-structural shocks.

Correlation being between -1 and 1, the indicator’s theoretical value will be between -2 and 2. But in practice, it will be hard to obtain an indicator higher than 1.5 or lower than -1.5. A higher absolute value indicates a more important mismatch.

In a static approach, Chart 1 below illustrates any possible theoretical position of the indicator with values of $\rho_r$ on the x-axis and those of $\rho_n$ on the y-axis, and allows to distinguish different zones according to the necessity of adjustments. For instance, the origin represents a situation in which the bilateral exchange rate between two currencies is completely flexible (thus the correlation of the two currencies is zero), and shocks are asymmetric, which corresponds to a zero correlation of shocks. Point A in the first quadrant reflects that the bilateral exchange rate is fixed and economic shocks are perfectly symmetric. Point C in the 3rd quadrant indicates that two countries variables change in a completely opposite sense. However, these three points (i.e., the origin, points A and C) are on the line connecting the two points A and C, on which each point represents a situation where exchange rate behavior is perfectly compatible with economic fundamentals. In other words, this line represents all points of equilibrium between nominal exchange rate co-movement and macro-structural shock symmetry. Then, this line can be called ‘zero-mismatch line’.
However, the region superior to the line, including point B as well as the entire 2\textsuperscript{nd} quadrant, represents ‘excessive co-movement zone’ and indicates a situation where the nominal exchange rate behavior should adjust, for instance, by appropriate policy decisions, towards less co-movement of nominal exchange rates. On the contrary, the region inferior to the line, including point D as well as the entire 4\textsuperscript{th} quadrant, may be called ‘excessive symmetry zone’ requiring adjustments towards more nominal exchange rate stability. This situation will justify for instance an exchange rate policy, which aims at strengthening the nominal exchange rate co-movement or fixity.

Combined with sliding-correlation approach, the indicator of mismatch shows evolutions of exchange rate co-movement and shocks symmetry and it allows to observe possible dynamic adjustment toward the zero-mismatch line. If the indicator moves horizontally, this means that it is characteristic of economic shocks that adjusts in function of exchange rate regime, and then we can say that there is a ‘real adjustment’. If the indicator moves vertically, this means that it is exchange rate behavior that adjusts in function of economic fundamentals. In this case, we can say that there is a ‘nominal adjustment’. But in practice, the indicator may also move diagonally, which can be defined as a ‘mixed adjustment’.

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In the last step of our analytical approach, this zero-mismatch-line-based discussion will be combined with synthesis focusing on positive or negative values of the indicator: important positive values will support arguments in favor of flexible exchange rate regime, while important negative values will support economic policies toward a fixed exchange rate regime or monetary integration.

3. Empirical results

For the empirical application, monthly and quarterly data are used on the UK, the USA, France and Germany for the period from 1970Q1 to 2008Q3. Monthly data on trade-weighted effective exchange rates are provided by the Bank for International Settlements, while quarterly data on real GDP and CPI are provided by IMF International Financial Statistics.

3.1. Behavior of nominal effective exchange rates

In order to see if the Sterling Pound behavior is more of the free-floating type or related to the US Dollar or to the currency unit of France and Germany, Chart 2 below compares the UK’s nominal effective exchange rate with those of the USA, France and Germany. In general, the GBP shows a co-varying trend with the USD, but a limited co-movement with the French currency and even an opposite behavior vis-à-vis the German currency. This mixed trend being checked in a more detailed way in the following section, one may observe in Chart 2 that the Sterling Pound is much more related to the USD than to French and German currencies (FF and then Euro / DM and then Euro)\(^9\) during the examined period. In particular, its co-movement with the USD was pronounced between the beginning of 1990s and the mid-2000s, while its opposite behavior compared to the European currencies was easily observable during the same period. The exit of the UK from the European Monetary System (EMS) and the recent US dollar crisis may denote, respectively, the beginning and the end of this period.

\(^9\) The effective exchange rate series of FF and DM are continued with the effective exchange rate of euro from 1999.
3.2 Estimated macro-structural shocks

For each of the examined countries, preliminary tests indicate that all variables of the standard VAR system are characterized as I(1) and they are not co-integrated with each other. The standard VAR model in first differences is then estimated by the OLS method using quarterly data from 1970Q1 to 2008Q3 for each of the examined countries, with four lags selected according to the AIC and SBC criteria. With these ‘first’ estimation results and using the structural VAR system, we identified the C_0 matrix and the series of structural shocks (AS, IS and LM shocks), for each of the four countries (UK, USA, France and Germany). Chart 3 below draws our estimates of these series.

3.3 Economic assessment of compatibility of the exchange rate regime of GBP with economic fundamentals

Using the series of nominal effective exchange rates and our estimates of macro-structural shocks described above, we compute UK’s indicators of mismatch regarding its relation vis-à-vis the USA, France and Germany.

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10 To check the robustness of our estimation results, including those of structural shock series in the following steps, we estimated the standard VAR with different lag lengths (between 2 and 8) selected by different criteria. But in terms of global significance level, we retained 4 lags for all variables and for all countries examined.
However, it should be pointed out that exchange rate behavior is characterized only by correlation of the nominal effective exchange rates, while characteristics of economic fundamentals can be measured by correlation of any of the three types of macro-structural shocks estimated. This means that we estimate three types of indicator of mismatch (i.e., AS shock-based, IS shock-based, and LM shock-based) for each of the three bilateral relations of the UK (i.e., vis-à-vis the USA, vis-à-vis France and vis-à-vis Germany).

All these results are given in the following Charts 4, 5a, 5b and 5c. Note that Chart 4 shows all the historical positions of the indicator plotted in the two-dimensional framework containing the diagonal zero-mismatch line, whereas specific (positive or negative) values of the indicator can be easily observable in Charts 5a, 5b and 5c.

In Chart 4, one can easily observe that all the three UK/US indicators are in the ‘excessive co-movement zone’ indicating that the GBP is too closely related to the USD while structural shocks are rather asymmetric. Even though there seems to be rather a nominal adjustment toward the zero-mismatch line, we can say that the GBP behavior is clearly disconnected from the UK’s economic fundamentals (in all the three macro-structural aspects) vis-à-vis the USA.

Sources: Author’s estimates using RATS and data from BIS & IMF
On the other hand, opposite results are shown in the UK/France and UK/Germany relations. Indicators for the relationships with France and Germany are in the ‘excessive symmetry zone’ indicating that the GBP is too ‘flexible’ given the similarity of economic fundamentals with its two partners from the Euro zone, and that UK should, in consequence, ‘correct’ the mismatching behavior of the GBP vis-à-vis the euro.

Charts 5a, 5b and 5c below show, in a more direct and specific way, evolutions of the UK’s nominal effective exchange rate co-movement, economic shock symmetry and values of indicator of mismatch. The mainly positive values of UK’s indicator of mismatch vis-à-vis the USA (Chart 5a) mean that the Sterling Pound has been co-varying with the US Dollar, while any type of its economic fundamentals was not in favor of a fixed exchange rate regime. On the contrary, the very significant negative values of UK’s indicator of mismatch vis-à-vis France (Chart 5b) and Germany (Chart 5c) show that the UK has been experiencing similar macro-structural shocks with these countries, and thus would recommend closer coordination of exchange rate policies and even to consider joining the Euro in order to reduce disconnection of its exchange rate regime from general economic conditions.
Chart 5a. UK's exchange rate co-movement, macro-structural shocks symmetry and indicator of mismatch vis-à-vis USA (1970 to 2008)

Sources: Author's estimates using RATS and data from BIS & IMF

Chart 5b. UK's exchange rate co-movement, macro-structural shocks symmetry and indicator of mismatch vis-à-vis France (1970 to 2008)

Sources: Author's estimates using RATS and data from BIS & IMF
4. Conclusion

Empirical evidence shows that UK, although its currency is apparently floating freely and displays significant instability in its relations with monetary units of its main European partners (Germany and France, and then presently with the Euro zone), do not display, as expected by OCA theory, asymmetries in macro-structural shocks (i.e. strong negative correlations).

This disconnection of the two measures suggests that floating of the Pound and high instability of its relations with the currency unit of its European partners is not justified by macro-structural characteristics. From an OCA approach point of view, there are no empirical evidences favoring the option of keeping UK out of the Euro zone, while ever closer economic links and growing instability at the world level are strong arguments in favor of the joining.

Symmetrically, apparent similarity in the evolution of UK and US effective exchange rates is not explained by significant symmetry of macro-structural shocks of the two countries.

There is no strong argument in favor of exchange rate policies targeted towards priority to stabilization with USD, targeting contradictory with the option of joining of UK with the Euro zone.

This paradoxical situation is probably to be explained by the stakes of the British financial sector, closely related to the global financial system where the USD still plays the dominant role. Yet closer financial integration with the Euro zone could enter in contradiction with this option of remaining disconnected with the Euro zone in terms of exchange rate. Of course this matter can only be

Sources: Author’s estimates using RATS and data from BIS & IMF
settled politically and issues such as debates over sovereignty can interfere with the logic of evaluation of costs and benefits of the two options.

References


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Appendix: Estimation of structural shocks

In order to recover the macro-structural shock series (decomposed here into AS shocks, IS shocks and LM shocks) to the UK, the USA, France and Germany, we use in this paper a structural VAR system. Following Blanchard & Quah (1989) and Clarida & Gali (1994), we estimate first the following 3-variable VAR (real GDP (GDP), real effective trade-weighted exchange rate (REX) and consumer price index (CPI)) with k lags, Δ and L being the usual difference and lag operators, respectively,

$$A(L)X_t = e_t \quad (A.1)$$

where: X is the vector of the 3 endogenous variables in first differences, i.e., $X_t = [\Delta GDP, \Delta REX, \Delta CPI]'$; $e_t$ is the vector of the estimated residuals in the standard VAR, i.e., $e_t = [e_{\Delta GDP}, e_{\Delta REX}, e_{\Delta CPI}]'$; with var($e_t$) = $\Omega$; $A_i$ are square 3x3 matrices of coefficients with $\sum_{i=0}^{k}A_i = I$ and $A_0 = I$. This model can be also written in a moving average form as:

$$X_t = B(L)e_t \quad (A.2)$$

where: $B_i$ are also square 3x3 matrices of coefficients with $\sum_{i=0}^{k}B_i = I$, and we have $B(L) = A(L)^{-1}$ and $B_0 = I$. In this reduced form, the endogenous variables are function of the residuals $e_t$ that can be easily estimated by the OLS method. But, if the endogenous variables are expressed as a function of white noise structural shocks $\varepsilon_t$, the VAR system can be written as:

$$X_t = C(L)\varepsilon_t \quad (A.3)$$

where: $\varepsilon_t$ is the vector of the white noise shocks, i.e., $\varepsilon_t = [\varepsilon_{AS}, \varepsilon_{IS}, \varepsilon_{LM}]'$ with var($\varepsilon_t$) = $I$ when normalised. Note that $\varepsilon_{AS}$ is the series of aggregate supply (AS) shocks, $\varepsilon_{IS}$ is the series of real demand (IS) shocks and $\varepsilon_{LM}$ is the series of monetary demand (LM) shocks; $C_i$ are the square 3x3 matrices of coefficients with $\sum_{i=0}^{k}C_i = I$, which define the dynamic structural relationship among the system’s 3 endogenous variables and the 3 white noise structural shocks.
In order to recover the series of macro-structural shocks ($\varepsilon_t$) from the data and the observed regression errors ($e_t$), it is sufficient to know the matrix $C_0$. This follows from the fact that, contemporaneously, the effect of the stochastic innovations $B_0e_t=e_t$ by necessity have to coincide with the effect of the white noise structural shocks $C_0\varepsilon_t$, i.e., $e_t=C_0\varepsilon_t\equiv S\varepsilon_t$, for every value of $t$ with a non-singular matrix $S$. This means that the identification of $C_0$ allows to identify and recover the series of $\varepsilon_t$ from the data and the observed errors $e_t$. Then, because $\Omega=ee'=(S\varepsilon_t)(S\varepsilon_t)'=SIS'$, we have

$$C_0C_0'=\Omega \quad (A.4)$$

To identify $C_0$ we need nine restrictions because $C_0$ contains nine unknown elements. Combined with the assumption of orthogonality and unit variance of the structural shocks $\varepsilon_t$, [A.4] provides six restrictions. We need now only three additional restrictions to exactly identify $C_0$. To do this, we use economic theory. In the Blanchard-Quah sense, the theoretical restriction is not imposed directly on the matrix $C_0$. In fact, as the VAR system is estimated in first-difference form, the effects of a shock on the level of a variable in the long run is represented by the sum of all coefficients of the structural shocks with lags. If $C_S$ refers to the matrix of these sums in the long run, we can say that $C_S=C_0+C_1+C_2+...+C_k$, where $k$ is the number of lags. If a shock $j$ has no long-run effect on the level of a variable $i$, it means that $C_S\{i,j\}=0$. As in Clarida & Gali (1994) as well as in many other open-economy macroeconomic models, we suppose that: (1) real demand shocks do not have any impact on real GDP in the long run; (2) monetary demand shocks do not have any impact on real GDP in the long run; (3) monetary demand shocks do not have any impact on trade-weighted real effective exchange rate in the long run. These restrictions imply, respectively, $C_S\{1,2\}=0$; $C_S\{1,3\}=0$; and $C_S\{2,3\}=0$. For example, this expression $C_S\{2,3\}=0$ means that LM shocks are supposed not to affect real effective exchange rate in the long run, but may have an impact in the short run. In other words, even though we suppose $C_S\{2,3\}=0$, we may have $C_0\{2,3\}\neq0$ indicating that there is an immediate impact of monetary demand shocks on real effective exchange rate variation. In the Blanchard-Quah sense, $C_S$ meets the following condition:

$$B_S\Omega B_S' = C_S C_S' \quad (A.5)$$

where $B_S$ and $\Omega$ are both obtained in the reduced form of the VAR system. Let us now define a new lower triangular matrix $H$ as the Choleski decomposition of $B_S\Omega B_S'$. As $C_S$ is also a lower triangular matrix, resulting from the theoretical restriction, we can say that:
CS = H \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \qu