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Firm entry and aggregate efficiency growth: An optimal dynamic Program of entry and R&D investment

Asma Raies*

Abstract:

The effect of entry on the aggregate efficiency growth is still theoretically and empirically unresolved. Many studies focused on this effect in short and long-run, without considering the dynamic transition and how do entry affect the convergence of the industry toward its long-run equilibrium? This paper aims to provide an answer and to fill this gap by employing optimal control principles. Our model exhibits saddle-path stability and shows that the effect of entry and entry liberalizing policy (reducing the entry cost) on the aggregate efficiency growth may be positive, negative or nil depending on the industry's initial characteristics (size and R&D). This theoretical result can justify the inconclusive current empirical evidence.

JEL: L11, L12, L22, L25, O41

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1. Introduction:

On the empirical level, the issue of the relative contribution of firms entry to aggregate productivity growth is still an object of controversies. Indeed, this contribution varies from one study to another, depending on the measurement of aggregate productivity, the time horizon over which changes occur, the business cycle, as well as on the country or industry under investigation. For example, *Baily et al. (1992)* and *Griliches and Regev (1995),* found that firm entry and exit had a small contribution to aggregate productivity growth for US manufacturing and Israeli industries respectively. According to *Foster et al. (1998),* the contribution of net entry to aggregate growth depends on the horizon over which the changes are measured. When high frequency data are used, the contribution of entry and exit to productivity growth is low, but with intermediate (a 5-year time horizon) or long run (a 10-year time horizon) data, the contribution of net entry is large. *Martin and Jaumandreu (2004)* find that entry has a positive and significant effect on Spanish aggregate productivity growth with a stronger impact in the period before Spanish integration in the EU. *Scarpetta et al. (2002)* analysed several OECD countries and found that the entry and exit contributed to between 20% to 40% of aggregate productivity growth. There were significant differences in the contributions of entry to aggregate productivity growth between Europe and the US. In the former, the entry of firms has a positive contribution to growth, but the effect is small, whereas in the latter, firm entry has a negative contribution to growth. Differences were also found in terms of the importance of the contribution to aggregate productivity growth across manufacturing sectors. In high technology sectors, the entry of new firms has a larger than average contribution to total growth. The results also differ according to whether aggregate productivity is measured by TFP or labour productivity, with a net

* College of Business, Umm Al Qura University, Saudi Arabia

entry having a strong contribution to TFP growth. *Brandt (2004)*, using a new data set covering nine EU countries suggests that high rates of firm entry coincide with rapid productivity, especially in the ICT related services sectors and in some business services industries, while in the more mature manufacturing industries, expenditure on formal R&D seems to be more important as a determinant of productivity growth. According to *Baldwin and Wulong (2006)*, firm entry explains 70 % of the aggregate labour productivity growth of the Canadian manufacturing sector over the period 1979-1999. Both *Toshiyuki and Kazuyuki (2005)* and *ITO Keiko (2011)*, using Japanese firm-level data respectively over the periods 1992-2002 and 2000-2007, found that the entry tends to be negatively associated with the productivity growth both in the manufacturing and the service sectors.

It comes out from these empirical divergences that the effects of entry liberalizing policies are still unresolved too. Indeed, *Srivastava (1996)* shows that the rate of TFP growth increased after deregulation in India in 1985. However, more recently, *Aghion, Burgess, Redding and Zilibotti (2005)*, provide evidence that deregulating entry in India in 1985 and 1991 has had an ambiguous effect. *Nicoletti and Scarpetta (2003)* and *Alesina et al. (2003)* found a negative effect of regulation reforms on productivity growth in OECD countries. This result is confirmed in *Brandt (2004)* for European Union. However, *Griffith and Harrison (2004)* show different impacts of entry liberalization on economic rent, R&D and growth rates of labour productivity and of TFP in the European Union over the period 1985-2000 as well as separately for the manufacturing and services sectors. The results obtained by *Cincera and Galgau (2005)*, using 352 digits sectors for 9 OECD countries suggest that the coefficient on regulation is allowed to differ across sectors with the sign and significance varying across sectors and countries. The result that product market reforms in different countries led to different experiences leads to the question of whether it is possible to impose a common structure across EU different countries.

To explain and justify such inconclusive current empirical evidence[1], the endogenous technological change literature provides a coherent and attractive framework for modelling efficiency growth at the aggregate level. Nevertheless, the effect of entry on the aggregate efficiency growth is still theoretically unresolved. That is by intensifying competition, the entry of new firms may either enhance or discourage the R&D activity of incumbent firms. Consequently, the growth rate of aggregate efficiency may either increase or decrease. Indeed, *Aghion et al. (2006)* provide one of the most recent models on the impact of firm entry or the threat of entry on incumbent firms' incentives to innovate, which in turn affects aggregate productivity growth. Firm entry or the threat of entry produces two effects on incumbents' incentives to innovate. On the one hand, there is an *escape entry effect or an encouragement effect* according to which, an increase in the threat of entry of new firms will increase the incentives to innovate in sectors that are close to the technological frontier because firms close to the frontier know that they can escape entry by new firms through innovation. On the other hand, there is also a *discouragement effect* of entry according to which, an increase in the threat of entry may discourage innovation in sectors that are initially far below their current

[1] The reader can refer to Duhautois et al. (2006), David Law and Nathan McLellan (2005) and Dean Parham (2002) for a negative effect of entry on productivity growth and to Hahn (2000), Jaan, Raul and Kaia (2004) and Admasu (2007) for a positive contribution of entry to aggregate productivity growth.

technological frontier. In this case, firms know that they are too far away from the frontier to win against a new entrant and they decrease innovation since the increase in the threat of entry leads to a decrease in the expected payoff from investing in R&D. The study shows too that liberalization (as measured by an increase in the threat of entry) encourages innovation in industries that are close to the frontier and discourages innovation in industries that are far from it. Productivity, output, and profits, should thus be higher in industries and firms that are initially more advanced. In the second part of the paper, the authors support empirically these evidences, using micro-level data for productivity growth and patenting activity for UK firms over the 1987-1993 period and from the liberalization experience in India over the 1990-1997 period. However *Aghion et al. (2006)*'s result is contrasted by some others empirical studies. For example, *Scarpetta et al. (2002)* found that tight product market regulations have a direct negative effect on productivity regardless of their position relative to the technology frontier. This negative effect is larger the further a country is from the technological frontier. This result is confirmed once again by *Nicoletti and Scarpetta (2003)* who show significant links between product market policies and productivity performance, with entry liberalisation leading to productivity gains in all of the OECD countries regardless of their position relative to the technology frontier. In addition, *Bettina (2006)* provides an endogenous growth model where entrants offer higher quality products than existing ones. This assumption implies a positive effect of entry (and of liberalizing entry policies) on the economic growth rate. This result is contrasted by empirical ones which found a negative effect of liberalizing entry on productivity growth. *Grossmann and Steger (2008)* conclude that if the incumbent firms do not benefit from entrants' R&D (absence of spillovers), the negative *discouragement effect* outweighs the *encouragement* one. If, on the contrary, technological spillovers exist, the net effect of entry becomes ambiguous. This theoretical result can't interpret empirical ones since technological spillovers exist in reality. *Carreira, C. and Teixeira, P. (2008)* provide a neo-Schumpeterian model to discuss how the mechanisms of entry and exit contribute to industry productivity growth in two alternative technological regimes. Their evolutionary approach assumes that individual firms learn about technology through a variety of sources (learning by doing, by using, by searching and learning from advances in science and technology and from inter-industry spillovers). The numerical simulations show that the industry-level productivity growth is higher in the entrepreneurial regime (where the improvements in the technological knowledge are mainly due to the new firms) than in the routinized regime (where such improvements are mostly associated with the established firms). More recently, *Acemoglu, D and Cao, D (2010)* find, despite the Schumpeterian character of their model, a strictly negative relationship between the rate of firm entry and the rate of aggregate productivity growth. This reflects the importance of the productivity growth by incumbents and the dominance of the *discouragement effect*. The resulting lower productivity growth by incumbents outweighs the higher growth due to entry. This result is contrasted by *Murao and Nirei (2011)* who develop an endogenous productivity growth model with heterogeneous innovation efficiency across firms. They show that a reduction in the entry cost decreases the equilibrium rent for innovators, and thus reduces the incentive for R&D with a stronger effect for high innovative types than low innovative types. Thus, the reduced entry cost may have a negative impact not only on the average R&D investments but also on the reallocation of market share from the low-innovative firms to the high innovative firm. By estimating the model on Japanese manufacturing firm level panel data over the period 2001-2004, they found the reduced entry cost greatly enhanced the positive entry effect, which outweighs the other two negative effects on the aggregate productivity growth.

All studies cited above are silent and represent a radical departure from the traditional theory of growth. However, they only focused on the effect of entry on aggregate efficiency growth in short and long-run and none of them considers the transition dynamics and how does entry affect the convergence of the industry toward his long-run equilibrium. We argue that there is much to learn by extending these models and consider the transition dynamics of these industries. This will improve our understanding of the phenomena and enables us to answer the question: why does entry contribute positively to aggregate productivity growth in some theoretical and empirical studies and negatively in others. The analytical framework we develop in this paper fills this gap. It studies the joint determination of the number of firms and the rate of productivity growth in a monopolistic competition model where many firms sell differentiated products; undertake cost-reducing R&D subject to a research technology characterized by dynamic increased returns at the firm level. We apply optimal control theory to firms' entry and R&D decisions. This allows us to study these two firms' behaviors which involve two important feed-back mechanisms. Indeed, the number of firms in the market determines the behavior of profit seeking firms because it determines the private benefits and costs of innovations. Thus the private and social benefits generated by tecnological progress and the performance of the economy vary with the number of firms as market rivalry varies. This (endogenous) number of firms in the market (entry decision) changes in response to demande and technological conditions which induces important feed-backs of technological process upon itself. In addition, by affecting the number of firms and their R&D decisions, entry of new firms influences the entrant's profit, and thus, the entry behavior itself. These feed-back mechanisms generate interdependance between the price, R&D investment, and entry decisions of firms which produces two opposite effects of entry on incumbents'R&D investment. On the one hand, there is a *rivalry effect* or an *encouragement effect* according to which an increase in the number of firms raises the incumbent firms' incentives to innovate and escape market rivalry. On the other hand, there is also a *discouragement effect* of entry according to which an increase in the firms number, lowers the incumbent firms' profit and thus reduces the return of R&D investment. By analysing the interactions between these opposit effects along the transition paths toward the equilibrium, our model shows that the economy converges to a stable saddle-point steady state with positive efficiency growth. This will happen if the initial number of firms is sufficiently low. By contrary, if market rivalry is very tough (the number of firms is very high), *the discouragement effect* outweighs the *encouragement one* bringing the economy back to zero-growth equilibrium. Finally, comparative statics show that the effect of entry liberalizing policy (reducing the entry cost) on the aggregate efficiency growth may be positive, negative or nil depending on the industry's initial characteristics (size and R&D).These theoretical results can justify the inconclusive current empirical evidence.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 determines the steady state equilibrium, discusses the properties of this steady state and analyses the transition dynamics of the model. Section 4 provides some comparative static results. Section 5 concludes.

2. The Model

We consider a closed economy with a fixed population *L* (normalized to 1) of identical households. The model describes the evolution of a market for a differentiated product in a continuous time. The industry consists of a continuum of firms producing each one a single, unique variety of the differentiated product. Therefore, the number of firms n_t equals the number of brands available to consumers.

2.1 Households

Households have symmetric preferences over a range of n differentiated goods. The preference ordering of identical consumers is described by the inter-temporal utility function:

$$U = \int_0^{+\infty} e^{-rt}\left(x_0(t) + Y_t\right) dt \tag{1a}$$

where $x_0(t)$ is the consumption of the numeraire in time *t*, and Y_t , is the consumption index of produced varieties of the Dixit-Stiglitz type:

$$Y_t = \left(\int_0^{n_t} (y_{j,t})^{\alpha} \quad dj \right)^{1/\alpha} \tag{1b}$$

where $y_{j,t}$ is the amount of variety *j* of the differentiated product demanded by a consumer at time *t*. Let *E* represents the total instantaneous expenditure on the differentiated products:

$$E = \int_0^{n_t} p_{j,t} y_{j,t} \quad d\,j \tag{2}$$

where $p_{j,t}$ is the price of variety *j* at time *t*.

Maximising Y_t , subject to (2), gives the demand function faced by firm *j* at time *t*, $y_{j,t}$:

$$y_{j,t} = \frac{p_{j,t}^{1/(\alpha-1)}}{\int_0^{n_t} p_{i,t}^{\alpha/(\alpha-1)} di} E \tag{3}$$

The demand function (3) is isoelastic with the elasticity of demand $\sigma = 1/(1-\alpha)$.

2.2 Incumbent firms

Each firm is endowed with one unit of labour that it devotes between production and R&D activities. This allocation of labour is endogenous. It is the result of the firm's R&D behavior that gives the amount of labour ($L^r_{j,t}$) devoted to the R&D activity. Thus ($L^p_{j,t} = L - L^r_{j,t}$) is the total amount of labour devoted by firm *j* to production.

The typical firm produces one differentiated consumption good with the technology:

$$L^p_{j,t} = c_{j,t}\, y_{j,t} \tag{4}$$

where $1/c_{j,t}$ is the labour productivity of firm *j*. We assume that[2] $w = 1$ so that $L^p_{j,t}$ is the total cost of production. Using the cost function (4), instantaneous firm *j*'s profit is:

$$\pi_{j,t} = (p_{j,t} - c_{j,t})\, y_{j,t} - L^r_{j,t} \tag{5}$$

where $L^r_{j,t}$ is the R&D expenditure.

The present value of net cash flow of firm *j* is given by:

[2] We assume that wage is constant (w=1) while labor productivity is increasing because there are sound theoretical reasons for predicting that there will be very little correlation between them. Indeed many theoretical and empirical researches provide strong support for this prediction. For example: Fixed-wage theory points out that wages are imperfectly correlated with productivities. (Aziariadis (1975 and 1983), Grossman (1981), polemarchakis and Weiss (1978),Killingsworth (1988)). Empirical tests (Abowd and Ashenfelter (1981) and Ham (1982))underline such facts. Wage-differentials literature empirically shows that heterogeneity of productivity fails to capture heterogeneity of wage among workers(Abowd and Ashenfelter(1981) and Kruegger and Summer(1988)).
For more recent empirical studies predicting no correlation between wage and productivity, the reader can refer to Bildirici (2004, 2005), Alp and Bildirici (2008), Harrison (2009) , Lopez-Villavicencio and Silva (2010), Fleck, Glaser and Sprague (2011), Mishel and Shierholz (2011), Greenhouse and Leonhardt (2006), Craig Simpson (2012) and many others.

$$V_{j,t} = \int_{t}^{\infty} e^{-r(\tau-t)} \pi_{j,\tau} \, d\tau \tag{6}$$

where r is the exogenous and constant interest rate.

Each incumbent firm acts as a monopoly in its own product market and chooses its optimal price by maximizing the present value (6) subject to the production technology (4) and the demand schedule (3). The optimal price strategy is therefore:

$$p_{j,t} = \frac{c_{j,t}}{\alpha} \tag{7}$$

Using (7), the firm j's profit becomes:

$$\pi_{j,t} = \frac{(1-\alpha)\hat{c}_{j,t} E}{\hat{C}_t} - L^r_{j,t} \tag{8}$$

where $\hat{c}_{j,t} = (c_{j,t})^{-\alpha/(1-\alpha)}$ and $\hat{C}_t = \int_0^{n_t} \hat{c}_{j,t} \, dj$

We call $\hat{c}_{j,t}$ "the firm j's efficiency''. Thus, the sector's average efficiency is $\hat{C}^m_t = \hat{C}_t / n_t$.

When profitable, firms establish in-house R&D facilities to produce a continuous flow of innovations and improve their efficiency level. Corporate R&D is described by the following dynamic relation:

$$\dot{\hat{c}}_{j,t} = \mu \hat{C}_t L^r_{j,t} \tag{9}$$

where $\dot{\hat{c}}_{j,t}$ is the flow of efficiency generated by a R&D project employing $L^r_{j,t}$ units of labour for an interval of time dt, and $\mu\hat{C}_t$ is the productivity of labour in R&D as determined by the exogenous parameter $\mu > 0$ and the stock of public knowledge measured by the sector's aggregate efficiency $\hat{C}_t$. The latter component captures the spillovers of knowledge across the firms and firms' inter temporal interaction in R&D. Indeed, each firm's efficiency increment adds to the aggregate efficiency and, hence, contributes to the efficiency of all other firms in the economy.

2.3 Entry

An entrepreneur must develop a new differentiated product and incur the fixed entry cost β to become an entrant and join the industry. We assume that potential entrants face the same entry cost β and they are, on average, as efficient as incumbent firms, i.e. their efficiency level is equal to the sector's average efficiency $\hat{C}_t^m$. This assumption can be justified by the fact that technological knowledge that was created by previous producers is a public good that each entrant may use in the production process of new ideas.

2.4 The economy's average efficiency growth rate

Recall that an increase in the total efficiency $\hat{C}_t$ may result either from the incumbent firms' R&D investments or from the entry of new firms. Analytically one can write:

$$\dot{\hat{C}}_t = \int_0^{n_t} \dot{\hat{c}}_{j,t}\, dj + \dot{n}_t \hat{C}_t^m \tag{10}$$

The growth rate of $\hat{C}_t$ is then:

$$\frac{\dot{\hat{C}}_t}{\hat{C}_t} = \frac{\int_0^{n_t} \dot{\hat{c}}_{j,t}\, dj + \dot{n}_t \hat{C}_t^m}{\hat{C}_t} \tag{11}$$

As $\dot{\hat{c}}_{j,t} = \mu \hat{C}_t L^r_{j,t}$, one can write:

$$\int_0^{n_t} \dot{\hat{c}}_{j,t}\, dj = \mu L_t^r \hat{C}_t n_t \tag{12}$$

where $L_t^r = \frac{1}{n_t} \int_0^{n_t} L^r_{j,t}\, dj$ stands for the average R&D investment of incumbent firms.

The equation (11) can be re-written as follows:

$$\frac{\dot{\hat{C}}_t}{\hat{C}_t} = \mu n_t L_t^r + \frac{\dot{n}_t}{n_t} \tag{13}$$

The economy's average efficiency is noted by:

$$\hat{C}_t^m = \hat{C}_t / n_t \tag{14}$$

Let $\hat{g}_t^m = \dot{\hat{C}}_t^m / \hat{C}_t^m$ be the growth rate of $\hat{C}_t^m$ which is given by the growth rate of $\hat{C}_t$ minus the entry rate, i.e.

$$\hat{g}_t^m = \frac{\dot{\hat{C}}_t}{\hat{C}_t} - \frac{\dot{n}_t}{n_t} \tag{15}$$

Replacing $\dot{\hat{C}}_t / \hat{C}_t$ by its expression given in equation (13) yields the average efficiency growth rate:

$$\hat{g}_t^m = \mu n_t L_t^r \tag{16}$$

2.5 Incumbent firm's R&D behavior:

The intertemporal problem of the incumbent firm is to maximize the value (7), subject to the R&D technology (9). The firm takes as given the rivals' innovation paths.

$$\underset{L^r_{j,t}}{Max}\ V_{j,t} = \int_t^\infty e^{-r(\tau - t)} \pi_{j,\tau}\, d\tau$$

$$subject\ to \quad \dot{\hat{c}}_{j,t} = \mu \hat{C}_t L^r_{j,t}$$

$$with \quad \pi_{j,\tau} = \frac{(1-\alpha)E\hat{c}_{j,\tau}}{\hat{C}_\tau} - L^r_{j,\tau}$$

The current-value Hamiltonian is:

$$H_{j,t} = \frac{(1-\alpha)\,\hat{c}_{j,t}\,E}{\hat{C}_t} - L^r_{j,t} + \lambda_{j,t}\,\mu L^r_{j,t}\hat{C}_t \tag{17}$$

where the costate variable, $\lambda_{j,t}$, is the value of the patent. The firm's efficiency level, $\hat{c}_{j,t}$, is the state variable and the R&D investment, $L^r_{j,t}$, is the control variable. Irreversibility of research and the finite labour supply impose bounds on the R&D investment, i.e.

$$0 \le \int_0^{n_t} (L^r_{j,t} + L^p_{j,t})\, dj \le L .$$

Maximizing the Linear Hamiltonian with respect to $L^r_{j,t}$ gives:

$$L^r_{j,t} = \begin{cases} 0 & \text{for} \quad 1 > \mu\lambda_{j,t}\hat{C}_t \\ L^r_t & \text{for} \quad 1 = \mu\lambda_{j,t}\hat{C}_t \\ L & \text{for} \quad 1 < \mu\lambda_{j,t}\hat{C}_t \end{cases}$$

where L^r_t is the singular solution characterized below. The case $1 > \mu\lambda_{j,t}\hat{C}_t$ implies that the value of the innovation is lower than its cost and the firm does not invest. The case $1 < \mu\lambda_{j,t}\hat{C}_t$ implies that the value of the innovation is higher than its cost and the firm invests L. The first order conditions for the interior solution are given by equality between the marginal benefit and the marginal cost of the innovation. This occurs when $1 = \mu\lambda_{j,t}\hat{C}_t$. The necessary conditions for interior optimum are:

- The derivative of $H_{j,t}$ with respect to $L^r_{j,t}$:

$$\frac{\partial H_{j,t}}{\partial L^r_{j,t}} = -1 + \lambda_{j,t}\,\mu\hat{C}_t = 0$$

Thus

$$\lambda_{j,t} = \frac{1}{\mu \hat{C}_t} \quad \Leftrightarrow \quad \frac{\dot{\lambda}_{j,t}}{\lambda_{j,t}} = -\frac{\dot{\hat{C}}_t}{\hat{C}_t} \tag{18}$$

- The derivative of $H_{j,t}$ with respect to $\hat{c}_{j,t}$:

$$\frac{\partial H_{j,t}}{\partial \hat{c}_{j,t}} = \frac{(1-\alpha)\ E}{\hat{C}_t} = r\,\lambda_{j,t} - \dot{\lambda}_{j,t} \tag{19}$$

By combining (18) and (19) we obtain the following first order condition:

$$(1-\alpha)\,\mu\ E - r = \dot{\hat{C}}_t\ / \hat{C}_t \tag{20}$$

This condition implies that the interior solution is the same for all firms. Hence, the firm *j*'s optimal R&D investment $L^r_{j,t}$ is equal to the sector's average R&D investment:

$$L^r_t = \frac{1}{n_t} \int_0^{n_t} L^r_{j,t}\ d\,j$$

Replacing $\dot{\hat{C}}_t / \hat{C}_t = \mu n_t\, L^r_t + \dot{n}_t / n_t$ in equation (20) yields:

$$L^r_t = \frac{(1-\alpha)\,\mu\ E - r - \dot{n}_t\ / n_t}{\mu\, n_t} \tag{21}$$

Equation (21) shows a negative relationship between the entry rate $\dot{n}_t\ / n_t$ and the average R&D investment L^r_t. However, variation in the number of firms n_t produces two opposite effects on the incumbent's R&D investment. On the one hand, there is a *rivalry effect* or an *encouragement effect* according to which, an increase in the number of firms n_t increases the incumbent firms' incentives to innovate and escape market rivalry. On the other hand, there is a *discouragement effect* of entry according to which, an increase in the firms number n_t lowers the average gross[3] profit, $(1-\alpha)E/n_t$, and thus reduces the average return of R&D investment.

2.6 Firms' entry behavior:

Entry at date *t* costs β and produces value V_t^e:

$$V_t^e = \int_t^{\infty} e^{-r(\tau-t)} \pi_\tau^e \; d\tau \tag{22}$$

where π_τ^e is the entrant's profit at date $\tau > t$.

In a free-entry equilibrium, $V_t^e = \beta$, which implies $\dot{V}_t^e = 0$.

By differentiating Equation (22) with respect to time, by using Equation (8), and the fact that $V_t^e = \beta$ and $\dot{V}_t^e = 0$, we get the following free-entry condition:

$$L_t^r = \frac{(1-\alpha)E}{n_t} - r\beta \tag{23}$$

2.7 Short-run aggregate equilibrium

Taking the time-derivative of the free-entry condition (23) yields the R&D dynamic equation $\dot{L}_t^r(n_t, L_t^r)$:

$$\dot{L}_t^r = -\frac{(1-\alpha)E}{(n_t)^2}\dot{n}_t \tag{24}$$

If we substitute the first-order condition (21) into Equation (24), we get the accumulation equation for L_t^r:

$$\dot{L}_t^r = -\frac{(1-\alpha)E}{n_t}\left(\mu\beta r n_t - r\right) \tag{25}$$

[3] The net average profit is: $\overline{\pi}_{j,t} = \frac{1}{n_t}\int_0^{n_t} \frac{(1-\alpha)\hat{c}_{j,t}E}{\hat{C}_t} - L_{j,t}^r \, dj = \frac{(1-\alpha)E}{n_t} - L_t^r$

The equilibrium of the economy is thus described by a system of two differential equations in the (L^r, n) space:

$$\begin{cases} \dot{n}_t = n_t\left((1-\alpha)\mu E - r - \mu n_t L_t^r\right) & (21) \\ \dot{L}_t^r = -\dfrac{(1-\alpha)E}{n_t}\left(\mu\beta r n_t - r\right) & (25) \end{cases}$$

subject to $\dot{n}_t \geq 0$ (i.e. subject to $n_t \leq \dfrac{(1-\alpha)\mu E - r}{\mu L_t^r}$)

The system formed by Equations (21) and (25) determines the path of evolution of n_t and L_t^r for a given initial values n_0 and L_0^r. Therefore, a conventional phase diagram will be drawn in terms of these two variables.

3. Steady state

We describe a steady state as a situation in which the number of firms n_t and the aggregate R&D investment L_t^r are constant so that the efficiency growth rate is constant too. Thus the steady-state values $n*$ and L^r* are determined by setting the expressions in Equations (21) and (25) to zero. This yields

$$n* = \frac{1}{\mu\beta} \text{ and } L^r* = \beta\left((1-\alpha)\mu E - r\right) \tag{26}$$

As we can see, at the steady state, a decrease in the entry cost β increases the number of firms $n*$ but reduces the average R&D investment L^r*. The latter effect offsets the former leading to a zero effect on the long-run aggregate efficiency growth rate. This growth rate rewrites at the steady state as follows:

$$g^m* = \mu n* L^r* = \mu E(1-\alpha) - r \tag{27}$$

3.1 Nature and stability of the steady state

To determine the nature and stability of this steady state equilibrium, we study the linear differential equation system that approximates (21) and (25) at L^r* and $n*$.

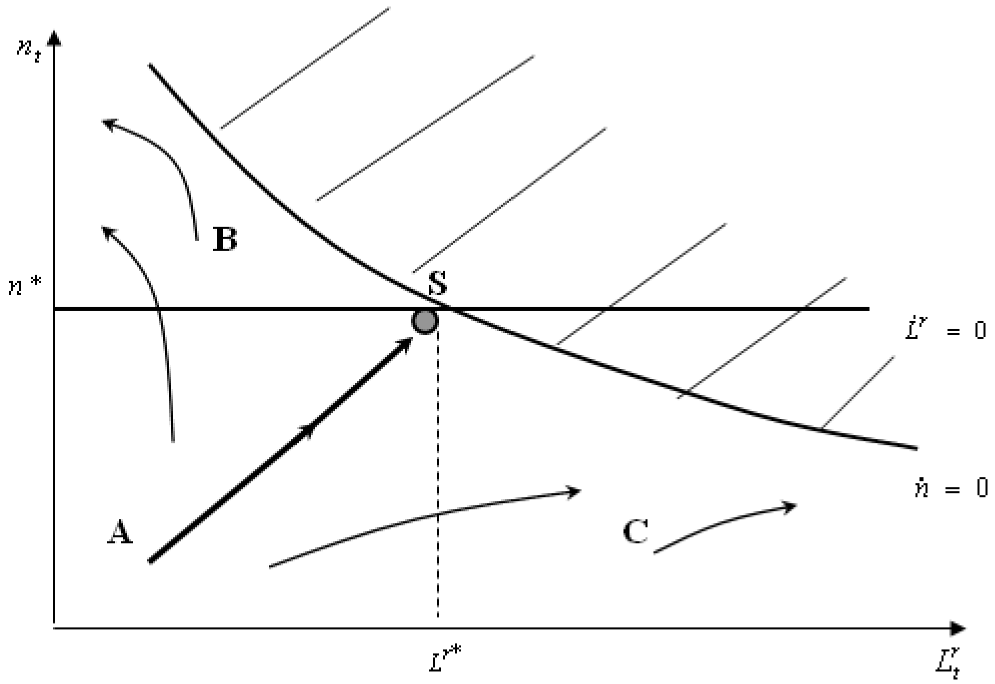
This consists on calculating the Jacobian matrix of the system and evaluating it at the steady state equilibrium. The eigenvalues of this Jacobian matrix determine the local stability properties of the economy. The Appendix 1 shows that the two eigenvalues are real numbers with one positive $(\lambda_1 > 0)$ and one negative $(\lambda_2 < 0)$.

This implies that the system converges monotonically to the stationary equilibrium which is a stable saddle-point.

3.2 Convergence and transitional dynamics

In this section, we characterize the transition dynamics of the model and discuss the main results of the article. We try to answer the question that motivated the beginning of this paper: why does firm entry enhance productivity growth in some empirical and theoretical studies and reduce it in others? The answer here is that the effect of firm entry on the average efficiency growth depends on both the initial industry's size (measured by n_t) and the R&D investment. This latter evolves in three possible regimes: singular R&D (at a rate $0 < L_t^r < L$), no R&D ($L_t^r = 0$) or maximal R&D ($L_t^r = L$). Equations (21) and (25) determine the path of n_t and L_t^r, for a given initial values n_0 and L_0^r. The phase diagram in Figure 1 shows the nature of the dynamics. We first display the horizontal axis in Figure 1 which corresponds to $n_t = n^* = \dfrac{1}{\mu\beta}$ and satisfies $\dot{L}_t^r = 0$ in (25). This equation implies that L_t^r is rising for n_t below this locus ($n_t < n^*$) and falling for $n_t > n^*$. We then display the downward sloping curve $n_t = \dfrac{(1-\alpha)\mu E - r}{\mu L_t^r}$ which shows combinations of n_t and L_t^r that satisfy $\dot{n}_t = 0$ in (21). Because firms do not exit the market, the portions of Figure 1 where $\dot{n}_t \leq 0$ are irrelevant. We can therefore confine the analysis to the region below the locus $\dot{n}_t = 0$, where n_t is rising ($\dot{n}_t \geq 0$). We can see on this figure that the curves $\dot{n}_t = 0$ and $\dot{L}_t^r = 0$ intersect in this region only at n^* and $L^r *$, which are the steady-state values. This intersection divides the space into three regions, A, B and C, which correspond respectively to the three R&D regimes discussed above. The arrows on the Figure 1 indicate the directions in which the processes evolve over time. The pattern of arrows is such that the economy can converge to the steady state if it starts in the region A and only if it is initially situated on the optimal transition pattern (which is unique). The other economies in the region A, will find itselves either in the region B or in the region C as indicated on this figure. Along the optimal transition path, both of the number of firms n_t and the average R&D investment L_t^r, increase monotonically toward the steady state. The transitional dynamics can be described as follows: the economy starts out in the no-R&D region where a few existing firms do not undertake in-house R&D. The profit made by these incumbent firms attracts entry. As more and more entrepreneurs enter the market, the industry becomes more competitive and incumbent firms begin investing in R&D to improve their efficiency levels. *The encouragement* or *rivalry* effect outweighs the *discouragement* one, thereby, the average R&D investment, L_t^r, increases. The economy then converges to a steady state where no new firms are created but where efficiency grows at a constant rate due to the R&D activity of the established firms.

Figure 1: The phase diagram



What happens to the economy's efficiency growth rate along this transition? The behavior of the growth rate $\hat{g}_t^m$ is given by (16). By replacing L_t^r given by (21), one can obtain the transition growth rate:

$$\hat{g}_t^m = (1-\alpha)\mu E - r - \frac{\dot{n}_t}{n_t} \tag{28}$$

Along the transition, the number of firms n_t increases. The behaviour of the growth rate, therefore, depends on the interaction between the *discouragement* and the *rivalry* effects. We can see from (28) that the growth rate is increasing in the number of firms n_t and decreasing in the number of entrants, $\dot{n}_t$, which means that $\hat{g}_t^m$ increases throughout the transition and approaches the steady state value from below. These theoretical results confirm the empirical ones which found a positive contribution of firm entry to aggregate productivity growth.

From the same figure, we can see that, in the region B, where initially the number of firms in the economy is over-adjusted (higher than, n^*), which means that the market rivalry is very tough, the negative *discouragement effect* outweighs the positive *encouragement one* causing the average R&D investment L_t^r to fall toward zero as n_t increases without bound and, bringing the economy back to a zero-growth situation$(\hat{g}^m = 0)$. This situation corresponds to the *no R&D regime* where firms do not invest in R&D because the value of the innovation is lower than its cost ($1 > \mu\lambda_{j,t}\hat{C}_t$). These theoretical results agree with the empirical studies suggesting a negative contribution of firm entry to the aggregate productivity growth.

The final possibility is that $n_t < n^*$ and $L_t^r > L^{r*}$ (region C). In this case, the initial average R&D investment is too high to remain in the saddle path. This situation corresponds to the *maximal R&D regime* where the value of the innovation is higher than its cost ($1 > \mu\lambda_{j,t}\hat{C}_t$). Both n_t and L_t^r rises infinitely but the economy diverges from the saddle point toward an unstable situation.

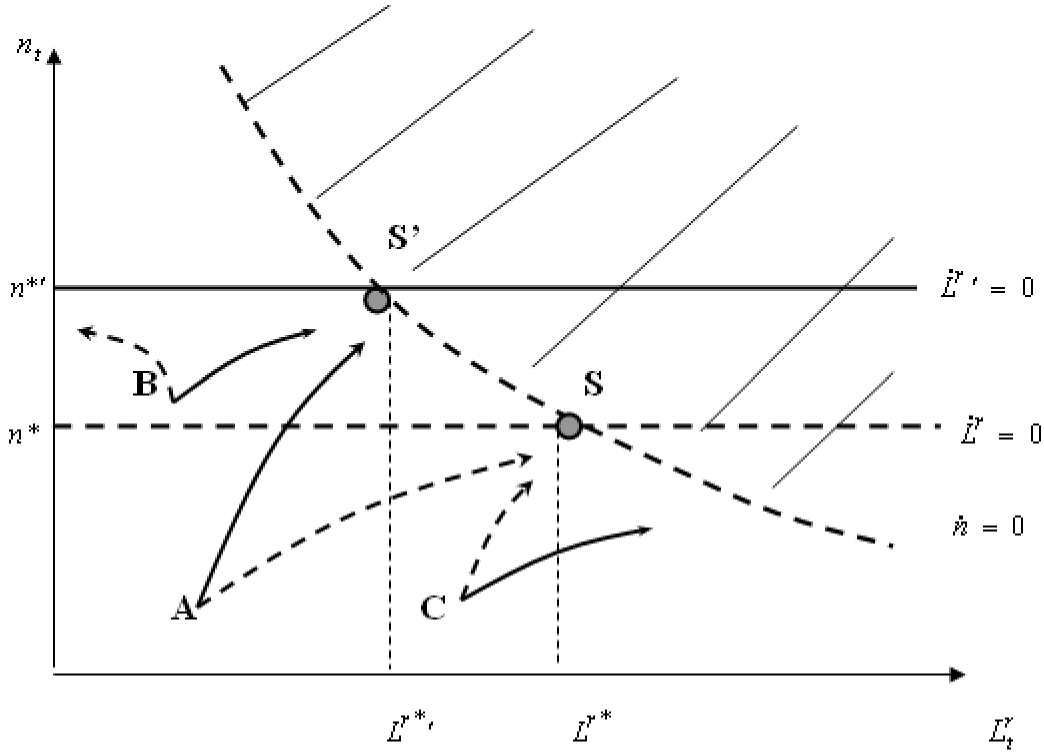
4. Comparative statics

4.1 The impact of entry liberalizing policy (reducing β)

We can use the phase diagram below (Figure 2) to analyze the long-run and the transitional effects of the entry liberalizing policy. As it can be seen, reducing the entry cost, β, will not affect the locus $\dot{n} = 0$ (represented by the dashed curves) but shifts up the locus $\dot{L}^r = 0$ (represented by the dashed horizontal axis). The new locus is represented by the solid horizontal axis ($\dot{L}^{r\prime} = 0$). The new intersection involves a new steady-state $S'(n^{*\prime}, L^{r*\prime})$ with a higher long-run number of firms ($n^{*\prime} > n^*$) and a lower R&D investment ($L^{r*\prime} < L^{r*}$). These effects arise because reducing the entry cost increases the incentives to enter the market which raises the number of firms n_t and thus discourages incumbent's R&D. As we can see from Equation (27), the latter effect offsets the former, which implies a zero effect on the steady state efficiency growth rate, g^{m*}.

Now, to answer the question why the effect of liberalizing entry policies is non-monotonic and varies across industries and countries, we distinguish three regions A, B and C characterized by different sizes n_t and/or average R&D investments, L_t^r. In each region, transition path before and after reducing β are shown by the dotted and solid arrows, respectively.

Figure 2: The impact of entry liberalizing policy (reducing β)



The figure shows that, before reducing the entry cost β, the industries situated on the optimal transition path, in the region A, tend to converge to the steady state $S(n^*, L^r{}^*)$. In this case, reducing β, will cause these industries to converge toward the new steady state $S'(n^{*\prime}, L^r{}^{*\prime})$ characterized by a bigger size ($n^{*\prime} > n^*$), a lower average R&D investment ($L^r{}^{*\prime} < L^r{}^*$), but the same growth rate $g^m{}^*$. These theoretical results can justify the empirical ones which found a zero and/or no significant effect of entry liberalizing policies on the aggregate productivity growth.

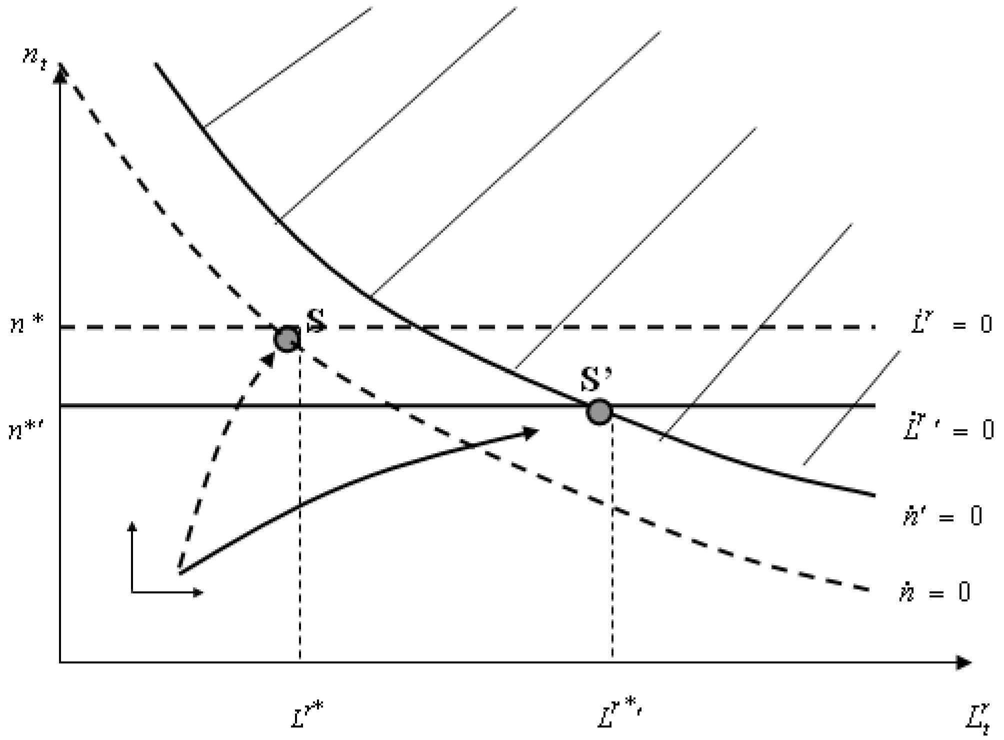
Before reducing the cost β, the industries situated in region B are diverging from the steady state $S(n^*, L^r{}^*)$ because of their over-adjusted size ($n^* < n_t < n^{*\prime}$). Thus, reducing β shifts-up the locus $\dot{L}^r = 0$, increases the steady state value of n_t from $n^* < n_t$ to $n^{*\prime} > n_t$, allowing these industries to converge to the new steady state $S'(n^{*\prime}, L^r{}^{*\prime})$. This theoretical result confirms the empirical ones suggesting a positive and significant effect of entry liberalizing policies on the aggregate productivity growth.

Finally, before reducing β, the industries situated in region C, (with a relatively high average R&D investment, $L^r{}^{*\prime} < L^r_t < L^r{}^*$) are in the neighborhood of the steady state S and may converge to it. Reducing β would cause them to move away from the saddle path before they reach it and diverge over time from S toward an unstable situation. This result confirms empirical studies suggesting a significant negative effect of entry liberalizing policies on the aggregate productivity growth.

4.2 The impact of improving R&D efficiency, μ

According to the dynamical system formed by Equations (21) and (25) and to the stationary levels given in (26) and (27), it follows that improving the R&D efficiency μ, has both transitional and long-run effects. Indeed, an increase in μ, lowers the $\dot{L}^r = 0$ locus and rises the $\dot{n} = 0$ locus. The new curves are represented in Figure 3 by $\dot{L}^{r\prime} = 0$ and $\dot{n}' = 0$, respectively. The new intersection involves the new steady state $S'(n^{*\prime}, L^{r*\prime})$ characterized by a lower number of firms $n^{*\prime} < n^*$ and a higher average R&D investment, $L^{r*\prime} > L^{r*}$. This arises because improved R&D efficiency enhances R&D of incumbent firms but discourages new entry. As can be deduced from Equation (27), the average efficiency growth rate g^{m*} increases in the long-run.

Figure 3: The impact of improving μ



5. Conclusion

This paper developed a dynamic model of firm entry and R&D investment with imperfect competition. These two firms' behaviors involve a feedback mechanism. Indeed, by affecting the number of firms and their R&D decisions, entry of new firms influences the entrant's profit, and thus, the entry behavior itself. The analytic framework allows us to study the transition dynamics of the economy. It shows that the economy starts out with a small range of consumption goods, each one supplied by a single firm. Households like variety and buy all available consumption goods. There is, therefore, a high return from bringing new goods to the market. Entry is costly, and entrepreneurs compare the present value of profits from introducing a new good to the entry cost. Once in the market, firms live forever and engage in price competition. When a sufficiently large number of firms have entered the market and rivalry has become sufficiently tough, incumbent firms invest in R&D in order to reduce costs, offer lower prices and steal market share. Finally, by investing in R&D, incumbent firms contribute to the pool of public knowledge and reduce the cost of future R&D. These intertemporal spillovers allow the economy to grow at a constant rate in the steady state. This is reached when entry peters out and the economy settles into a stable industrial structure.

In addition, the phase diagram, shows that when initially the number of firms in the economy is very high,which means that market rivalry is very tough, the negative *discouragement effect* outweighs the positive *rivalry effect* and the average R&D investment decreases. The economy then diverges from the steady state toward zero-growth equilibrium where an infinite number of existing firms don't undertake in-house R&D. Finally, the comparative statistics analysis shows that the effect of entry liberalizing policy (reducing the entry cost) on the aggregate efficiency growth may be positive, negative or nil depending on the industry's initial characteristics (size and R&D) which can justify the inconclusive current empirical evidence.

Appendix

To determine the nature and stability of the steady state equilibrium, we study the linear differential equation system that approximates (21) and (25) at L^r* and $n*$. Indeed, linearizing around the steady state permits to study the dynamic and stability of a non-linear system of equations by transforming it in to a linear one. This consists on calculating the Jacobian matrix of the system and evaluating it at the steady state equilibrium. This linearization yields the following system:

$$\begin{bmatrix} \dot{L}_t^r \\ \dot{n}_t \end{bmatrix} = \begin{bmatrix} 0 & \Omega_n \\ \Psi_{L^r} & \Psi_n \end{bmatrix} \begin{bmatrix} L_t^r - L^r* \\ n_t - n* \end{bmatrix}$$

Where the elements of the Jacobian matrix evaluated at the steady state are:

$$\Omega_n = \frac{\partial \dot{L}^r}{\partial n} = -\frac{\mu \beta r E(1-\alpha)}{n*} < 0 \qquad ; \qquad \Psi_{L^r} = \frac{\partial \dot{n}}{\partial L^r} = -\mu(n*)^2 < 0$$

and $\Psi_n = \dfrac{\partial \dot{n}}{\partial n} = -\big((1-\alpha)\mu E - r\big) < 0$

The eigenvalues of the Jacobian matrix determine the local stability properties of the economy. These eigenvalues are defined as the roots of the characteristic equation given by $\det(\lambda I - J) = 0$, where λ is an eigenvalue, J is the Jacobian matrix and 'det' is the determinant of the matrix $(\lambda I - J) = 0$. In the steady state, the characteristic equation writes:

$$\lambda^2 - \Psi_n \lambda - \Omega_n \Psi_{L^r}$$

Which have the following roots:

$$\lambda_1 = \frac{\Delta^{1/2} + \Psi_n}{2} > 0 \qquad \text{and} \qquad \lambda_2 = -\frac{\Delta^{1/2} - \Psi_n}{2} < 0$$

Where $\Delta = (\Psi_n)^2 + 4\,\Omega_n \Psi_{L^r} > 0$

The two eigenvalues are real numbers with one positive $(\lambda_1 > 0)$ and one negative $(\lambda_2 < 0)$. This implies that the system converges monotonically to the stationary equilibrium which is a saddle-point.

The different solutions to an arbitrary pair of linear differential equations may exhibit the following patterns:

a) *The eigenvalues are real and of opposite signs:* the stationary point is a stable saddle point.
b) *The eigenvalues are real and both positive:* the path of the differential equations cannot converge to steady state. It will move away from it.
c) *The eigenvalues are real and both negative:* the equilibrium would be completely stable with all paths converging to it.
d) *The eigenvalues are complex with negative real part:* the path would show transitory oscillations until they reach the steady state, which is stable.
e) The eigenvalues are complex with positive real part: the path moves away the steady state with transitory oscillations.

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